THE CRAWL OF THE CRAB (Computer-based Version) 2

PLEASE READ PRINTED VERSION
FOR INTRODUCTION

Special Instructions for the Computer-Based Version

To perform this exercise, you will need enough computers for your students (one per team) and image display software. The Microsoft Office spreadsheet application Excel or equivalent will also be helpful – details are listed below. If you do not meet these requirements, you will need to use the printable version of this exercise.

1) Computers

Either PCs or Macintoshes can be used for this exercise. Since this exercise involves measuring small objects on the screen, the larger the monitor the better. It is recommended that you have at least a 15" monitor with 800x640 resolution (the higher the resolution the better).

2) Image Display Software

The exercise involves measuring the distance between features in the images of the Crab Nebula. You will need image display software that reports the cursor position in pixels to allow the coordinates of the features to be recorded by the students. A variety of free software is available that will accomplish this. We recommend GIMP (***www.gimp.org***) or ImageJ (rsbweb.nih.gov/ij/), free software that you can download and install on the computer relatively easily. The packages are small and can be downloaded and installed in a few minutes on most any platform. If you prefer to use your own software, make sure that the position of the cursor is displayed to an accuracy of one pixel. Any lower accuracy *will not work* for this exercise.

3) Spreadsheet

An Excel spreadsheet custom-made for this exercise is included with this Educators Guide CD. However, students can record their answers on the Student Worksheet if necessary. Using Excel is therefore optional, but if you do not have access to it we recommend using the printable version of this exercise. If you do not have Excel but wish to use the worksheet, a free program called Open Office (***www.openoffice.org***) will also be able to open the spreadsheet and will run on most platforms. A version of Open Office designed to run natively on the Mac, called NeoOffice, is also available for free (***www.neooffice.org***).

Materials for each team of 2 or 3 students:

- Computers (one per student team)
- Imaging software
- Digital images of the Crab Nebula from 1956 and 1999 (supplied)
- Blank Excel Spreadsheet (supplied)
- Graph paper (optional; plots can be made using Excel)
- Student Handout and Worksheet (one per student, supplied below)

Procedure:

1) Pre-class: *Make sure you meet the requirements for this exercise as outlined in the Special Instructions for the Computer-based Version section above*. Read through the Background Information section, and go over the Student Handout and Worksheet. Students should work in teams of 2 or 3, so print out enough Handouts and Worksheets for the students. Prepare the computers as outlined in the Special Instructions for the Computer-based Version section above.

2) In class: With the students, go over information about supernova remnants provided in the Background Information section. Show them the two Crab Nebula images (they are on the CD; the filenames are crab_1956.jpg and crab_1999.jpg), and tell them that by simply measuring the changes between two images, they will be able to determine the age of an astronomical object. Also on the CD is an animated image (crab_expansion_animation.gif) which shows the expansion of the nebula over time. Showing them this will help them understand what it is they will be measuring.

Go over any vocabulary (such as "knot", etc.). Before they start to make their measurements, stress that not all the knots are easy to measure, so they should be careful, and they should also pay particular attention to any tips given in the exercise. If the knots cannot be seen well in the images, advise the students to change the contrast and brightness levels on the images.

The Excel spreadsheet will be used to calculate the distances between the knots and the pulsar, the rates of change, and the age of the supernova remnant. These calculations can be tedious, and it's easier on the student to let the spreadsheet make the calculations. However, the students will learn more if you modify the spreadsheet so that they do some of the calculations by hand using the worksheet.

3) Post-class: With the class, go over the students' results, and compare them to the "true" age of the Crab. How many students were close, how many were way off? Discuss possible places where errors could creep in, including general methods used, or particular knots which may have caused problems. Tell them that methods such as the ones they used are also used by real scientists to find the ages of many astronomical objects.

Answer Key - "The Crawl of the Crab"

The answers the students get will depend on how they measured the features in the nebula. As an example, however, we have supplied with the CD an Excel sheet (crawlofthecrab_answers.xls) with typical answers on it.

Supplemental Activity

Plotting and Linear Regression

The default version of the activity computes the age of the remnant by averaging ages derived from each individual knot. While this method is simple conceptually, it is not the one likely to give the most reliable result. Instead, we can use a least-squares fitting method called **linear regression** to fit a line to the data. Excel and most other popular spreadsheet programs have built-in ability to do this. The method should be less sensitive to outlier points than a straight average. Below we outline how you can have your students do this if you desire. We also give a reference where you can learn how the method works if you like.

To use the linear regression feature, you should ask the students to use the additional cells marked "Distance" and "Velocity" on the right side of the spreadsheet. As with the rest of the sheet, the cells come programed to grab the details they need; you can remove the programming in these cells and have the students put it in themselves if you like.

The spreadsheet will calculate the slope of the best-fit line through the points. These are indicated in the table at the far-right, at the top of the spreadsheet. The correlation coefficient *(r)* is also computed. Note that the built-in functions *slope()* and *rsq()*, valid in NeoOffice, are used to compute these values. The same or similar functions are available in Excel and standard Open Office (as well as other spreadsheet programs); just use the Help menu of your software to find the corresponding functions (search under *linear regression*). If you would like to learn how the linear regression method works, consult a book such as *Numerical Recipes* (Press, W. H., Teukolsky, S. A., Vettering, W. T. and Flannery, B. P., Cambridge University Press, 3rd Edition) or similar reference.

Once the slope and correlation coefficient have been found, the students are ready to find the age of the remnant and the date of the explosion. The age is found by taking the reciprocal of the slope. The explosion date can be found by subtracting the age from 1999. These operations are also set up in the default spreadsheet. Again, you might want to modify the spreadsheet so that the students do the calculation by hand. Alternatively, they can program the appropriate formulas into the spreadsheet themselves.

It is instructive to have the students plot the data and best-fit line. This is fairly easy to do in Excel when you know how. Have the students switch to the Graph spreadsheet (select the tab at the bottom of the Data spreadsheet). To plot the data, first select all the data to be plotted. If you select the cells with the labels along with the data cells, then these labels will be used for the plot axes automatically (usually). To plot the data, do the following:

1. Select boxes b2 to c12
2. Click on the Chart button in the menu bar (or select Chart in the Insert Menu)
3. The chart wizard will appear. Select Next.
4. Select the Scatter Plot chart type (perhaps also called xy chart). Make certain that the data series is selected to be in columns. Select Next.
5. Select the plot of data points only, with no connecting lines. Select Next.
6. For Axis Titles, select both x and y axes. Be sure the appropriate titles are in the boxes. Write them there if they are not (x is Distance, y is Velocity). Select Create. Say yes if asked to sort the x-data.
7. Resize the chart by grabbing its corners and dragging so that it is easily readable.
8. Double click inside the chart to bring it into Edit Mode.
9. Place the mouse over one of the data points, then left click to focus the points (they will become highlighted).
10. Right click on any point to get a pop-up menu If using Excel, a Trendline option will be offered at this point. Select it. You are done. In NeoOffice or OpenOffice, select Object Properties.
11. In NeoOffice or OpenOffice, select the Statistics panel from the pop-up menu.
12. Choose the Trendline item in the Regression Curves box, and then click Okay. A best-fit line will be plotted through the points.

Incidentally, you might have noted that we are plotting the same variables that Hubble did for galaxies. Essentially, we are making a "Hubble diagram" for the supernova remnant. Since we assume (to first approximation) that the remnant expands at constant rate, it will follow a Hubble relation (to first order), just as the universe does (also to first order).

Things to Consider

1. How does the age found using linear regression compare to the age found using averages?

Ans: Our answers are fairly close, with the linear regression model doing a slightly better job. Both answers are younger than the actual age though.

2. Are there any points in the first method that deviate by large amounts from the average? If so, what effect to you think they might have? Can you think of a way to minimize the effects of such points? (Hint: you could simply drop them from your analysis, but that would bias your result, which is generally considered a bad thing to do... In addition to the mean, you might think about using the median or mode).

Ans: There are several points that deviate quite a lot from the average. However, there are about as many above and below the mean, and the deviations are roughly symmetrical on either side. One way to characterize the quality of the mean is to use the stddev() function to find the standard deviation (we get 116 years, or about 14%). You might have the students compute these values for their data, which is likely to be slightly different because of differences in how they measure individual knots. The median might be a better way to estimate the age, as it is less sensitive to outliers. However, in this case the median age (833 years) is about the same as the mean. The mode is not useful with only ten data points.

3. We did not use the y-intercept in our analysis. What do you expect the y-intercept to be for this system? How could you use the y-intercept function in Excel to perform a "sanity check" for your analysis.

Ans: The y-intercept should ideally be zero since the remnant begins as a star which has effectively zero size. The intercept we get is about 0.01, which is "close" to zero. If we got something very different from zero we might suspect the quality of our fit was poor.

4. Excel can be used to plot your data. Your teacher can help you do this if you don't know how. You can also plot your best-fit line on top of the data. Create such a plot to see graphically what the data and fit look like.

THE CRAWL OF THE CRAB - 2
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Introduction

Two images of the Crab Nebula supernova remnant, taken 43.75 years apart, clearly show the expansion of the explosion. In this exercise, you will determine the age of the Crab by measuring how much it has expanded over that period of time. You will convert the amount of expansion to a rate of expansion, and from there work backwards to determine the date the star exploded, the birthday of the Crab Nebula.

Procedure:

First, examine both images. They are presented in grey scale (what most people erroneously call "black and white"), and are reversed such that bright objects like stars are black, while dark objects like the background sky appear white. This is an old astronomer's trick to make faint detail easier to see. You can see that the gas is not smooth; there are filaments and knots of gas strewn throughout the nebula.

One image was taken in February 1956, and the other in November 1999. The images appear similar at first glance, but if you look carefully you'll see some differences. The nebula itself has changed during the time interval between the two images. It is this change that you will measure, and from that determine when the Crab was born.

Some tips before you begin:

a) Getting a useful cursor shape: In most image display packages, the shape of the cursor depends on what tool you are using (cropping, pen, shapes, etc.). Some of these make it easier to see your cursor position than others. It might be easier to read the cursor position by clicking on different tools such as the crop or pen tool. There is no "correct" tool, so just try different tools to see which one works best for you. In GIMP, the cursor shape default is a paintbrush that is a filled-in black circle 11 pixels across.

b) How to read the cursor pixel position in GIMP:

The position of your cursor in pixels is displayed in the lower left corner of the screen. If you click on the point, it will mark it with the shape chosen by the cursor. Marking a point in this way will allow you to refer to it later and try to remeasure it.

c) c) The Excel spreadsheet with this lab will be used to calculate distances between objects in the images for you. It can also be used to complete the entire activity, including the supplemental exercise (depending on how your instructor has arranged things). Make sure you record all the calculations you do by hand on the student worksheet.

d) It will be easier to do a careful job of measuring the knot positions if you enlarge the image to at least 100%.

Part I)

On both images, there are 10 knots of gas marked. Starting with the image from 1956, carefully measure the X and Y coordinates of each knot. Repeat these measurements for the 1999 image. (Note: you can open both files at once in GIMP.) Some of the knots are extended, or spread out a bit. For knots like that, pick an obvious feature to measure, like the center of the knot or the edge on one side. Make sure you pick the same feature in both images! If you don't, your results will not be accurate. The position of the pulsar has already been filled in for you in the spreadsheet – check to make sure that your measurements agree before filling in values for the knot positions.

Useful Tips: measure each knot in both images before going on to the next knot, rather than measuring all the knots in one image and then in the other. That way, you can be more consistent in the way you measure each knot. Another tip: it might help to measure the knots on the 1999 image first since it has better resolution and shows the structure of the knots more clearly. A third tip: sometimes measuring to the edge of a knot is easier than measuring to the center.

Part II)

The Excel spreadsheet will automatically calculate the distances from the pulsar for the various knots using the following formula (compliments of Pythagoras):

$$\textbf{Distance} = ((\text{X position of knot} - \text{X position of pulsar})^2 + (\text{Y position of knot} - \text{Y position of pulsar})^2)^{1/2}$$

The following parts may or may not be provided in the Excel spreadsheet, depending on the choice of your instructor:

Part III)

The Excel spreadsheet calculates the exact number of years that have passed between the time the first and second images were taken. The spreadsheet will also calculate the expansion rates for every individual knot using the following formula:

(Distance from pulsar in 1999-Distance from pulsar in 1956)/ Elapsed time

Part IV)

The Excel spreadsheet calculates the age of the nebula for each individual knot based on the following equation

Age = Distance from pulsar in 1999/ Expansion rate

Final)

The Excel spreadsheet gives you the average calculated age of the supernova remnant and the year which it exploded. How does this compare to the actual observed year of 1054 AD?

Supplemental Activity

Student Handout

If your teacher assigns the supplemental exercise, then you will use the technique of **linear regression**, to calculate the age of the remnant. This involves fitting a line through a set of x-y points. Since we are assuming that the supernova remnant has expanded at a constant rate, we expect that the distance traveled by individual knots is given by the relation s=vt, where s is the distance traveled, v is the velocity, and t is the time to travel the distance s. This equation describes a line. On a plot of distance vs. time, the slope of this line is the speed traveled. Below we explain how to use linear regression to estimate the age of the remnant.

Most spreadsheet programs have built-in functions that perform linear regression. In Excel and Open Office one such function is *slope()*, which gives us the slope of the best fit line through our data. There will also be a function that computes the correlation *coefficient, r*, for the fit, and another that computes the y-intercept of the line. In Open Office the function for the correlation coefficient is *rsq()*. The coefficient gives us a rough (very rough) idea about how good the line fits our data. Very crudely put, a value of *r* near 1 means a good fit, and a value of *r* near 0 means a poor fit (this has been greatly over-simplified, but regression analysis is beyond the scope of this exercise). The spreadsheet employs these two functions to find the slope of the line best fitting your data, as well as the correlation coefficient for the fit. The default spreadsheet does not compute the y-intercept, but your teacher might have you program it and/or the other functions into the cells yourself.

To solve our problem, we can consider finding the slope of a line with velocity as a function of distance. In other words, we can write the distance relation as $v = s/t$. Written this way, the slope is $1/t$, so the age of the remnant is the reciprocal of the slope. To get the year of the explosion, simply subtract this age from the year that the second image was taken, 1999. The default spreadsheet does these computations for you, but your teacher might have modified it somewhat.

Things to Consider

1. How does the age found using linear regression compare to the age found using averages?
2. Are there any points in the first method that deviate by large amounts from the average? If so, what effect to you think they might have? Can you think of a way to minimize the effects of such points? (Hint: you could simply drop them from your analysis, but that would bias your result, which is generally considered a bad thing to do... In addition to the mean, you might think about using the median or mode).
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